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Inflation in Brans–Dicke Theory with Torsion

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We show that an inflationary phase may occur at a sufficiently early epoch in the Robertson–Walker universe model in Brans–Dicke theory with torsion. Some features of this inflationary scenario are briefly discussed.

1. INTRODUCTION

Inflation is important because it is thought that it can solve such cosmological problems as horizons, homogeneity and flatness, and the magnetic monopole (Abbott and Wise, 1984a–c). It is well known that in the RW universe model filled with an ideal fluid with pressure p and energy density ρ, the scale factor $a(t)$ satisfies the Friedman equation

$$\ddot{a} = -4\pi(\rho + 3p)/3.$$

The dot denotes the derivative with respect to the cosmic time t. The inflationary condition $\ddot{a} > 0$ implies then the appearance of a negative effective pressure $p < -1/3$, which can only be achieved in this scenario with a dominant vacuum contribution to the total stress-energy tensor (Guth, 1981). Berman and Som (1989) showed that an inflationary phase may occur in the RW universe for the Euclidean case in Brans–Dicke theory with torsion, under the positive pressure condition. Gasperini (1986) showed that the spin density of the matter source can cause an inflation. The spin angular kinetic energy can also cause an inflation (Bradas, 1987). We will show that an inflationary epoch may occur in Brans–Dicke theory with torsion.

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2. FIELD EQUATIONS

We adopt the RW metric

$$ds^2 = dt^2 - a^2(t)[dr^2/(1-kr^2) + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2] \qquad (1)$$

where $a(t)$ is a scale factor.

We introduce vierbein fields V^i_μ

$$\begin{aligned} V^0_\mu &= (1, 0, 0, 0), \qquad V^1_\mu = (0, a/(1-kr^2)^{1/2}, 0, 0) \\ V^2_\mu &= (0, 0, ar, 0), \qquad V^3_\mu = (0, 0, 0, ar \sin\theta) \end{aligned} \qquad (2)$$

where the Latin letters denote the anholonomic coordinates and Greek letters denote the holonomic coordinates. Considering the 1-form $\omega^i = V^i_\mu\, dx^\mu$, we have

$$ds^2 = \eta_{ij}\omega^i\omega^j$$

Now we start with the following Lagrangian density for BD theory with torsion:

$$\mathscr{L} = (-g)^{1/2}(-R\phi + \omega\phi_\mu\phi^\mu/\phi + 16\pi L_m) \qquad (3)$$

where R is the curvature scalar constructed from the connection Γ^k_{ij}, ϕ is the BD scalar field, and $\phi_\mu = \partial\mu\phi$; L_m is the Lagrangian density of matter. For a spinless Lagrangian, Jha *et al.* (1988) obtained the following equations:

$$\phi G_{ij} = -(8\pi T_{ij} + \omega X_{ij}/\phi) \qquad (4)$$

$$\phi T^k_{ij} = \phi_{,i}\delta^k_j - \phi_{,j}\delta^k_i \qquad (5)$$

$$\Box\phi = 4\pi T/\omega \qquad (6)$$

where

$$X_{ij} = \phi_{,i}\phi_{,j} - \tfrac{1}{2}\eta_{ij}\phi_{,k}\phi^{,k}$$

$$T^k_{ij} = F^k_{ij} - F_j\delta^k_i + F_i\delta^k_j$$

$$F_i = F^j_{ji}, \qquad T = T^i_i$$

and F^k_{ij} are the torsion tensors, and T^k_{ij} are the modified torsion tensors. We consider an isotropic and homogeneous universe; then the scalar field is only dependent on time. From equation (5) we have the nonzero components of the torsion

$$F^1_{10} = F^2_{20} = F^3_{30} = \dot{\phi}/2\phi \qquad (7)$$

For convenience, we denote $h = \dot{\phi}/2\phi$.

From the first and second Cartan equations

$$\tfrac{1}{2}F^i_{jk}\omega^j \wedge \omega^k = d\omega^i + \omega^i_j \wedge \omega^j$$

$$d\omega^i_j + \omega^i_k \wedge \omega^k_j = \tfrac{1}{2}R^i_{jkf}\omega^k \wedge \omega^f$$

we get the nonzero components of the connection 2-form:

$$\omega_1^0 = \omega_0^1 = (h + \dot{a}/a)\omega^1, \qquad \omega_0^3 = \omega_3^0 = (h + \dot{a}/a)\omega^3$$
$$\omega_2^0 = \omega_2^0 = (h + \dot{a}/a)\omega^2, \qquad \omega_1^3 = -\omega_3^1 = (1 - kr^2)^{1/2}\omega^3/ar$$
$$\omega_1^2 = -\omega_2^1 = (1 - kr^2)^{1/2}\omega^2/ar, \qquad \omega_2^3 = -\omega_3^2 = \operatorname{ctg}\theta\omega^3/ar$$

and the nonzero components of the curvature tensors:

$$R_{101}^0 = R_{001}^1 = R_{002}^2 = R_{003}^3 = [(ah)_{,t} + \ddot{a})/a = A$$
$$R_{121}^2 = R_{131}^3 = -R_{223}^3 = k/a^2 + (h + \dot{a}/a)^2 = C$$

The nonzero components of the Einstein tensors are

$$G_{00} = -3C, \qquad G_{11} = G_{22} = G_{33} = 2A + C$$

We assume that the matter is an ideal fluid

$$T_{00} = \rho, \qquad T_{11} = T_{22} = T_{33} = p, \qquad T_i^i = \rho - 3p$$

where

$$T_{ij} = T_{\mu\nu} V_i^\mu V_j^\nu, \qquad T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg$$

Finally, the field equations (4) and (6) become

$$3k/a^2 + 3(h^2 + 2h\dot{a}/a + \dot{a}^2/a^2) = 8\pi\rho/\phi + \omega\dot{\phi}^2/2\phi^2 \tag{8}$$
$$k/a^2 + (h + \dot{a}/a)^2 + 2[(ah)_{,t} + \ddot{a}]/a = -8\pi p/\phi \tag{9}$$
$$\ddot{\phi} + 3\dot{a}\dot{\phi}/a = 4\pi(\rho - 3p)/\omega \tag{10}$$

3. DISCUSSION AND CONCLUSION

From equations (8) and (9), we have

$$[(ah)_{,t} + \ddot{a}]/a = -4\pi(1 + 3\gamma)\rho/3\phi - \tfrac{1}{3}\omega h^2 \tag{11}$$

Substituting equation (10) into equation (11), we get

$$\ddot{a}/a = -4\pi\rho[(1 + 3\gamma)/3 + (1 - 3\gamma)/2\omega]/\phi + (2 - \omega/3)h^2 + 2h\dot{a}/a \tag{12}$$

Equation (8) may be rewritten as

$$(\dot{a}/a)^2 = 8\pi\rho/3\phi + (2\omega/3 - 1)h^2 - 2h\dot{a}/a - k/a^2 \tag{13}$$

We can neglect the terms $2h\dot{a}/a$ and k/a^2 in equations (12) and (13); they are simplified as

$$\ddot{a}/a = -4\pi\rho[(1 + 3\gamma)/3 + (1 - 3\gamma)/2\omega]/\phi + (2 - \omega/3)h^2 \tag{14}$$
$$(\dot{a}/a)^2 = 8\pi\rho/3\phi + (2\omega/3 - 1)h^2 \tag{15}$$

Combining equations (14) and (15), we get a conservation equation

$$\frac{d}{dt}\left[\frac{8\pi\rho'}{3}+\left(\frac{2\omega}{3}-1\right)h^2\right]=2\frac{\dot{a}}{a}\left[-4\pi\rho'\left(1+\gamma+\frac{1-3\gamma}{2\omega}+3h^2-\omega h^2\right)\right] \quad (16)$$

where we have taken $p=\gamma\rho$, $\rho'=\rho/\phi$.

Letting $\rho'=Ba^n$ and $h^2=Ha^m$, where m and n are some numbers to be determined and B and H are constants, it is easy to obtain that

$$n=-3[1+\gamma+(1-3\gamma)/2\omega], \qquad m=-6(\omega-3)/(2\omega-3) \quad (17)$$

Now we can calculate the terms neglected in equations (12) and (13). If we choose $\omega<3/2$, we have $m<-3$. For the sufficiently early epoch of the evolution of the universe, we should have $a \ll 1$, so the following relations can be satisfied:

$$2h\dot{a}/a \ll (2-\omega/3)h^2, \qquad k/a^2 \ll (2\omega/3-1)h^2 \quad (18)$$

Considering equation (14), one immediately obtains that an accelerated expansion ($\ddot{a}>0$) can take place in the case of positive pressure even if the condition

$$(2-\omega/3)h^2 > 4\pi\rho[(1+3\gamma)/3+(1-3\gamma)/2\omega]/\phi \quad (19)$$

is satisfied.

From the inflationary conditions $\ddot{a}>0$ and $\dot{a}^2>0$, we have the corresponding minimal value and maximal value of a, respectively:

$$a_{\min}=[(3-2\omega)H/8\pi B]^{1/[6(\omega-3)/(2\omega-3)-3-3\gamma-3(1-3\gamma)/2\omega]} \quad (20)$$

$$a_{\max}=\left\{\frac{(2-\omega/3)H}{4\pi B[(1+3\gamma)/3+(1-3\gamma)/2\omega]}\right\}^{1/[6(\omega-3)/(2\omega-3)-3-3\gamma-3(1-3\gamma)/2\omega]} \quad (21)$$

A physically interesting inflationary scenario is characterized by the inflation factor $Z=a_{\max}/a_{\min}$ sufficiently high, $Z \geq 10^{30}$ (Guth, 1981). In our model this condition can be satisfied. For instance, if we choose $\omega=1/2$ and $\gamma=2/3-\epsilon$, $10^{-165}>\epsilon>0$, and the above inflation may occur.

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